GIIT PROFESSIONAL COLLEGE

(Affiliated to KOLHAN UNIVERSITY, Chaibasa)

Question Bank			**Course**	:	**BSc IT 1st Year**
Subject Code	:	**ITS01**	**Subject**	:	**MATHEMATICS**
All questions carry equal marks.					

Calculus

1. If $y = e^{ax} \sin bx$, find y_n

2. If $y = \dfrac{1}{x^2 + a^2}$ find y_n

3. State and prove Leibnitz's Theorem. (n-th derivation of the product of two functions)

4. If $y^{\frac{1}{m}} + y^{\frac{1}{m}} = 2x$, prove that $(x^2 - 1)\, y_2 + xy_1 - m^2y = 0$, where $y_1 = \dfrac{dy}{dx}$, $y_2 = = \dfrac{d^2y}{dx^2}$

5. If $y = \dfrac{Sin^{-1}x}{\sqrt{1-x^2}}$, $|x| < 1$, show that

    i. $(1 - x^2)y_2 - 3xy_1 - y = 0$
    ii. $(1 - x^2)y_{n+2} - (2n + 3)xy_{n+1} - (n + 1)^2y_n = 0$

6. If $y = \cos(10 \cos^{-1} x)$, show that $(1-x^2)y_{12} = 21xy_{11}$

7. If $y = \cos(m \sin^{-1} x)$, Show that

    i. $(1 - x^2)y_2 - xy_1 + m^2y = 0$
    ii. $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y = 0$

    Also, find the value of y_n when $x = 0$

8. If $y = e^{\cos-1x}$, Show that an equation connecting y_n , y_{n+1} and y_{n+2} is given by $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + 1)y_n = 0$

9. If $y = \sin^{-1} x$, then show that

    i. $(1 - x^2)y_2 - xy_1 = 0$
    ii. $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$

    Find also the value of $(y_n)_0$

10. If $\log y = \tan^{-1}x$, then prove that

    i. $(1 + x^2)y_2 + (2x - 1)y_1 = 0$
    ii. $(1 + x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n+1)\, y_n = 0$

GIIT PROFESSIONAL COLLEGE

(Affiliated to KOLHAN UNIVERSITY, Chaibasa)

Question Bank		**Course**	:	**BSc IT 1st Year**
Subject Code	: **ITS01**	**Subject**	:	**MATHEMATICS**

All questions carry equal marks.

11. State Maclaurin's series Infinite from.
12. State Rolle's theorem- Expansion of function in Infinite power series. Taylor's serious (extended to infinity)
13. Expand $(\sin^{-1}x)^2$ in a series of ascending power of x.
14. Assuming expansion of sin x , prove that

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

From the series $$\text{Sin x} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

which converges for all vales of x, we get the required result by differentiation.

15. Show that the maximum value of xy subject to the condition

    $3x + 4y = 5$ is $\frac{25}{48}$

16. When does the function sin3x - 3sinx attain its maximum or minimum values in $(0, 2\pi)$?
17. Show that of all rectangles of given area, the square has the smallest perimeter.
18. Show that the maximum value of $x^2 \log\left(\frac{1}{x}\right)$ is $\frac{1}{2e}$.
19. Prove that the function $f(x, y) = x^3 + 3x^2 + 4xy + y^2$ attains a minimum at the point

    $\left(\frac{2}{3}, -\frac{4}{3}\right)$.

20. Find the extreme value of $f(x, y) = 2x^2 - xy + 2y^2 - 20x$.

GIIT PROFESSIONAL COLLEGE

(Affiliated to KOLHAN UNIVERSITY, Chaibasa)

Question Bank		**Course** :	**BSc IT 1st Year**
Subject Code :	**ITS01**	**Subject** :	**MATHEMATICS**

All questions carry equal marks.

Vectors

1. If $\vec{r} = \vec{a}\cos wt + \vec{b}\sin wt$, show that

i. $\vec{r} \times \frac{d\vec{r}}{dt} = w\,\vec{a} \times \vec{b}$ *ii.* $\frac{d^2\vec{r}}{dt^2} = -w^2\,\vec{r}$

Where $\vec{a}$ *and* $\vec{b}$ *are constant vectors*

2. If $\vec{r_1} = t^2\,\vec{i} - t\,\vec{j} + (2t+1)\,\vec{k}$

$\vec{r_2} = (2t - 3)\,\vec{i} + \vec{j} - t\,\vec{k}$,

Find (i) $\frac{d}{dt}(\vec{r_1} \cdot \vec{r_2})$ (i) $\frac{d}{dt}(\vec{r_1} \times \vec{r2})$ when t = 1

3. If a is a unit vector, prove that

$$\left|a \times \frac{d\vec{a}}{dt}\right| = \left|\frac{d\vec{a}}{dt}\right|$$

4. If $\hat{r}$ is the unit vector in the direction of $\vec{r}$, show that $r \times d\hat{r} = \frac{\vec{r} \times d\vec{r}}{r^2}$

5. If $F = \frac{\vec{r} \times \vec{a}}{\vec{r} \cdot \vec{a}}$ where **a** is a constant vector, find $\frac{d\vec{F}}{dt}$**.**

6. If $\vec{a} = \sin\Theta\,\vec{i} + \cos\Theta\,\vec{j} + \Theta\,\vec{k}$

$\vec{b} = \cos\Theta\,\vec{i} - \sin\Theta\,\vec{j} - 3\,\vec{k}$

$\vec{c} = 2\,\vec{i} + 3\,\vec{j} - \vec{k}$

find $\frac{d}{d\theta}\{\vec{a} \times (\vec{b} \times \vec{c})\}$ at $\Theta = 0$

7. A particle moves along the curve **x = a cos t**, **y = a sin t** and **z = bt,** find velocity and acceleration at **t = 0** and **t = π/2.**

GIIT PROFESSIONAL COLLEGE

(Affiliated to KOLHAN UNIVERSITY, Chaibasa)

Question Bank		**Course** :	**BSc IT 1st Year**
Subject Code :	**ITS01**	**Subject** :	**MATHEMATICS**
All questions carry equal marks.			

8. If **a** and **b** are constant vector and t the time variable, show that a particle whose position vector at any instant is

$$\vec{r} = \vec{a}\cos wt + \vec{b}\sin wt$$

is moving in an ellipse whose centre is the origin and that the motion is due to a central force varying as the distance.

9. Evaluate $\frac{dy}{dx}\left(\vec{r}\,.\frac{d\vec{r}}{dt} \text{ x } \frac{d^2\vec{r}}{dt^2}\right)$

10. Show the necessary and sufficient condition for the vector $\vec{V}$ of the scalar variable **t** to have constant magnitude is

$$\vec{V}.\frac{d\vec{V}}{dt} = 0$$

11. If $\vec{V} \text{ x } \frac{d\vec{V}}{dt} = 0$, then Show that $\vec{V}(t)$ is a constant vector i.e $\vec{V}(t)$ has a fixed direction.

12. Find the value of $\vec{r}$ satisfying the equation

$$\vec{a} \text{ x } \frac{d^2\vec{r}}{dt^2} = b;\ (\vec{a}\,.\,\vec{b} = 0)$$

13. If $\vec{r} = a\cos t\,\vec{i} + a\sin t\,\vec{j} + t\,\vec{k}$, find $\frac{d\vec{r}}{dt}, \frac{d^2\vec{r}}{dt^2}$ and $\left|\frac{d^2\vec{r}}{dt^2}\right|$

14. If $r = t^2 i - t j + (2t + 1) k$, find the value of

   i. $\frac{d\vec{r}}{dt}.\frac{d^2\vec{r}}{dt^2}$ ii. $\left|\frac{d\vec{r}}{dt}\right|$ iii. $\frac{d^2\vec{r}}{dt^2}$ at $t = 0$

15. If $\vec{r_1} = t^3\vec{i} - t^2\vec{j} + t\vec{k}$ and $r_2 = (t+1)\vec{i} + (t+2)\vec{j} - 3t\vec{k}$, find

   i. $\frac{d}{dt}(\vec{r_1}\,.\,\vec{r_2})$ ii. $\frac{d}{dt}(r_1 \text{ x } r_2)$ at $t = 2$

16. If $\vec{r} = \vec{a}e^{nt} + \vec{b}e^{-nt}$ where a, b are constant vectors, show that

$$\frac{d^2 r}{dt^2} - n^2 r = 0$$

17. Given $\vec{r} = 4a \sin^3\theta\, i + 4a \cos^3\theta\, j + 3b \cos 2\theta\, \vec{k}$, prove that

$$\left(\frac{dr}{d\theta} \times \frac{d^2 r}{d\theta^2}, \frac{d^3 r}{d\theta^3}\right) = -216\, a^2 b \sin^3 2\theta$$

18. A particle moves along a curve whose parametric equation are $x = e^{-1}$, $y = a \cos 3t$, $z = b \sin 3t$ where t is the time and a and b are constant scalars.

i. Determine its velocity and acceleration at any time.

ii. Find the magnitudes of velocity and acceleration at t = 0.

19. If $\frac{d\vec{a}}{dt} = c \times a$, $\frac{d\vec{b}}{dt} = \vec{c} \times \vec{b}$, show that $\frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{c} \times (\vec{a} \times \vec{b})$

20. Prove that

$$\frac{d}{dt}\left\{\vec{u} \times \frac{d\vec{v}}{dt} - \frac{d\vec{u}}{dt} \times \vec{v}\right\} = \left\{\vec{u} \times \frac{d^2\vec{v}}{dt^2} - \frac{d^2\vec{u}}{dt^2} \times \vec{v}\right\}$$

21. Evaluate the derivatives of the following w.r.t. t.

$$\frac{\vec{r} + \vec{a}}{\vec{r}^2 + \vec{a}^2}$$

22. Evaluate $\frac{d^2}{dt^2}\left(r \cdot \frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right)$

23. Evaluate $\frac{d}{dt}\left\{\left(r \times \frac{d\vec{r}}{dt}\right) \times \left(\frac{d^2\vec{r}}{dt^2}\right)\right\}$

24. If $\vec{v} \cdot \frac{d\vec{v}}{dt} \times \frac{d^2\vec{v}}{dt^2} = 0$, show that $v \times \frac{d\vec{v}}{dt}$ has a fixed direction and that **v** is parallel to a fixed plane.

25. Grad $(\emptyset \pm \Psi)$ = grad $(\emptyset \pm grad\, \Psi$

i.e. $\nabla(\emptyset \pm \Psi) = \nabla\emptyset \pm \nabla\Psi$

26. Div $(\vec{a} \pm \vec{b})$ = div $\vec{a} \pm$ div $\vec{b}$

i.e. $\nabla(a \pm b) = \nabla . a \pm \nabla . b$

27. Curl $(\vec{a} \pm \vec{b})$ = curl $\vec{a} \pm$ curl $\vec{b}$

i.e. $\nabla(a \pm b) = \nabla \times a \pm \nabla \times b$

28. Grad $(\emptyset\Psi)$ = $\emptyset$ grad $\Psi + \Psi grad\ \emptyset$

i.e. $\nabla(\emptyset\Psi) = \emptyset\nabla\Psi + \Psi\nabla\emptyset$

29. $\nabla\left(\frac{\emptyset}{\Psi}\right) = \frac{\Psi\nabla\emptyset - \emptyset\nabla\Psi}{\Psi^2}$

30. Curl $(\emptyset\,\vec{a}) = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times (\emptyset\,\vec{a})$

i.e. $\nabla(\emptyset\,\vec{a}) = \emptyset(\nabla \times \vec{a}) + (\nabla\emptyset) \times \vec{a}$

31. Div $(\vec{a} \times \vec{b}) = \vec{b}$. (curl $\vec{a}$) – $\vec{a}$. (curl $\vec{b}$)

i.e. $\nabla(\vec{a} \times \vec{b}) = \vec{b}.(\nabla \times \vec{a}) - \vec{a}.(\nabla \times \vec{b})$

32. Curl $(\vec{a} \times \vec{b}) = \nabla \times (\vec{a} \times \vec{b}) = (\vec{b} . \nabla)\vec{a} - (\vec{a} . \nabla)\vec{b} + \vec{a}$ div $\vec{b} - \vec{b}$ div $\vec{a}$

33. $\nabla \times (\nabla\emptyset) = 0$ i.e curl (grad $\emptyset$) = 0

34. Div (curl $\vec{v}$) = 0 i.e $\nabla . (\nabla \times \vec{v}) = 0$

35. Find the unit vector normal to the surface $z^2 = x^2 + y^2$ at the point (-1, -2, 5)

36. If $\vec{V} = x^2yz\,\vec{i} + xy^2z\,\vec{j} + xyz^2\,\vec{k}$, find

i) div $\vec{V}$ ii) curl $\vec{V}$ iii) curl curl $\vec{V}$

37. Find curl $\vec{V}$, where $\vec{V} = e^{xyz}(\vec{i} + \vec{j} + \vec{k})$

38. Find div (crul $\vec{F}$) where $\vec{F} = x^2y\,\vec{i} + xz\,\vec{j} + 2yz\,\vec{k}$

39. If $\vec{F} = (x + y + 1)\,\vec{i} + \vec{j} - (x + y)\,\vec{k}$, show that $\vec{F}$. curl $\vec{F} = 0$

GIIT PROFESSIONAL COLLEGE

(Affiliated to KOLHAN UNIVERSITY, Chaibasa)

Question Bank			**Course**	:	**BSc IT 1st Year**
Subject Code	:	**ITS01**	**Subject**	:	**MATHEMATICS**

All questions carry equal marks.

40. Find div V and curl V where $V = \nabla (x^3 + y^3 + z^3 - 3xyz)$

41. Evaluate i) $\nabla . \vec{r}$ ii) $\nabla \times \vec{r}$

42. Evaluate ∇r^m

43. Evaluate $\nabla^2 (r^m)$

44. Prove that $\text{div}(\text{grad } r^m) = \nabla . (\nabla r^m) = m(m+1)r^{m-2}$

45. If $\vec{r}$ be a position vector and a, b are constant vector, prove that

i) $\text{div}[(\vec{r} \times \vec{a}) \times \vec{b}] = 2\, \vec{b} . \vec{a}$

ii) $\text{curl}[(\vec{r} \times \vec{a}) \times \vec{b}] = \vec{b} \times \vec{a}$

iii) $\vec{a} . \nabla \left(\vec{b} . \nabla \frac{1}{r}\right) = \frac{3(\vec{a} . \vec{r})(\vec{b} . \vec{r})}{r^2} - \frac{\vec{a} . \vec{b}}{r^3}$

46. Prove that

$\text{div}(u\nabla v) - \text{div}(v\nabla u) = u\nabla^2 v - v\nabla^2 u$

i.e $\nabla . (u\nabla v - u\nabla u) = u\nabla^2 v - v\nabla^2 u$

47. Find $\nabla\emptyset$, if

i) $\emptyset = \log(x^2 + y^2 + z^2)$

ii) $\emptyset = x \sin z - y \cos z$

iii) $\emptyset = r^2 e^{-r}$

iv) $\emptyset = x^2 + y - z - 1$ at the point (1, 0, 0)

48. Find $\nabla . \vec{F}$ where

i) $\vec{F} = 4x^2 \vec{i} + 3xy \vec{j} + 9z^2x \vec{k}$

ii) $\vec{F} = x^2z \vec{i} - 2y^3z^2 \vec{j} + xy^2z \vec{k}$

iii) $\vec{F} = (x^2 - y^2) \vec{i} + 2xy \vec{j} + (y^2 - xy) \vec{k}$

49. Find the curl of the vectors

$x^2z \vec{i} - 2y^3z^2 \vec{j} + xy^2z \vec{k}$ at the point (1, -1, 1)

GIIT PROFESSIONAL COLLEGE

(Affiliated to KOLHAN UNIVERSITY, Chaibasa)

Question Bank		Course :	BSc IT 1st Year
Subject Code :	ITS01	Subject :	MATHEMATICS

All questions carry equal marks.

50. If r = x i + y j + z k, find r . ∇∅ where ∅ = $x^3 + y^3 + z^3 - 3xyz$

51. Find div F and curl $F = x \cos z\, i + y \log x\, j - z^2 \vec{k}$

52. Find the unit vector normal to the surface $x^2y + 2xz = 4$ at the point (2, -2, 3)

53. Show that div grad $\left(\tan^{-1}\frac{y}{x}\right) = 0$

54. Compute $\nabla^2 r, \nabla^2 r^2, \nabla^2(r^{-2})$, where $r = \sqrt{x^2 + y^2 + z^2}$

55. Prove that

$$\nabla^2\left[\nabla.\ \left(\frac{\vec{r}}{r^2}\right)\right] = \frac{2}{r^4}$$

56. If $\vec{\boldsymbol{a}}$ is constant vector, prove that

$$a\,.\,\nabla.\ \left(\frac{1}{r}\right) = \left(\frac{a\,.r}{r^3}\right)$$